

University of Manitoba
Department of Mathematics
**MATH 8810 High-Dimensional Convexity:
Covering, Illumination, and Borsuk’s Question**
Winter 2027

PIMS network course — hybrid delivery

Instructors and course information

Instructors	Dr. Andrii Arman and Dr. Andriy Prymak
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Calendar course	MATH 8810, Topics in Geometry 1 (3 credit hours)
Lectures	Tuesdays and Thursdays, 11:30 a.m.–12:45 p.m. (Winnipeg local time)
Location	University of Manitoba: Armes 115; online: Zoom
Course webpage	https://prymak.net/coveringW27/
Office hours	By appointment, in person or via Zoom.
Course dates	The first class meeting is Thursday, January 7, 2027, and the last Tuesday/Thursday meeting is Thursday, April 8, 2027. There are no classes during the Winter Term Break, February 15–19.

Email policy. Students should use their official institutional email. Include “MATH 8810” in the subject line and your name, student number, and home institution in the body of the message. The instructors normally check email at least once per business day. All students are expected to check their institutional email and the course webpage regularly.

Course emphasis and learning objectives

This graduate-level course explores extremal properties of convex bodies in high dimensions. It covers the Hadwiger–Boltzmann illumination conjecture and Borsuk’s partition question, with a special focus on classical geometric covering theorems, probabilistic methods, discretization, and additive combinatorics. Other themes include bodies of constant width, volume bounds, and Lebesgue’s universal covering problem.

By the end of the course, a successful student should be able to:

- formulate the principal covering, illumination, and diameter-partition problems and explain the relationships between them;
- state and prove selected classical results and describe their high-dimensional consequences;
- apply probabilistic methods, discretization, fractional and hypergraph coverings, and additive combinatorics to geometric covering problems;
- explain key properties and constructions of bodies of constant width, including their roles in illumination and Borsuk’s question;
- read and critically assess a research proof, distinguishing established results from open questions; and
- communicate a mathematical argument clearly in writing and in an oral presentation, including answers to questions about its details.

Prerequisites

Consent of the instructors based on a review of the student's academic background. Students are expected to have a rigorous undergraduate foundation in linear algebra and vector geometry, real analysis, and introductory probability. Foundational concepts specific to convex geometry will be reviewed throughout the course to ensure a common baseline.

Format and participation across universities

The course will be delivered in a hybrid format for students at the University of Manitoba and other participating universities. Lectures will be streamed live via Zoom, using on-screen writing tools for real-time mathematical exposition. Zoom access information will be provided to enrolled students.

Students located at UM will attend and participate in person in the lecture room (TBA). For other participating PIMS network sites with more than one enrolled student, the host institution will be asked to arrange a dedicated room with videoconferencing equipment to facilitate group participation. Remote students may ask questions and participate in discussion during the live lectures.

All lectures will be recorded by the instructors. Screen recordings and accompanying written notes will be made available to all enrolled students for review and reference.

The teaching schedule follows the University of Manitoba's Winter 2027 calendar. Students registered through another university should check their own registration, withdrawal, grading, and program requirements. If a home-institution calendar or policy creates a conflict, contact the instructors and the local course coordinator early.

Texts and readings

The course draws on research articles, surveys, and selected chapters from the references listed at the end of this outline. Main background references include R. Schneider, *Convex Bodies: The Brunn–Minkowski Theory* [21]; H. Martini, L. Montejano, and D. Oliveros, *Bodies of Constant Width* [17]; and M. Naszódi's survey on translative coverings [18]. Precise reading assignments, lecture notes, and additional recent papers will be announced on the course webpage. Students are not required to purchase a textbook.

The exact topics will also depend on current research progress, which is rapid in these areas, including through AI-assisted proofs. Recent results and preprints may therefore be incorporated into the course, with updated readings announced during the term.

Evaluation of student performance

Component	Weight
Midterm exam (in person, two hours, evening of March 9, 2027)	30%
Final exam (in person, two hours, during final examination period April 13-25, 2027)	30%
Virtual presentation: design and mathematical content	20%
Virtual presentation: delivery and answers to follow-up questions	20%

Test/exam. Students will be asked to formulate definitions and/or theorems, give a short proof of a result covered in class, and solve short questions based on exercises provided during lectures. Both midterm and final examinations have the same format. The dates, times, and locations will be announced as soon as these details are available. Students at other universities will write the tests in person under local supervision, coordinated with their home institution.

Virtual presentation. Each student will give a presentation on a topic relevant to the course and agreed

upon with the instructors. The presentation assesses mathematical understanding, organization and exposition, delivery, and responses to follow-up questions. Presentation dates and detailed expectations will be announced during the term.

Exercises, feedback, and missed assessments

Exercises will be provided during lectures to support learning and preparation for the tests. Marked work and feedback will normally be returned within one week of the relevant assessment. Midterm exam results will be made available before the UM voluntary withdrawal deadline of March 22, 2027.

Students who cannot attend an assessment because of illness or other circumstances should contact the instructors as early as possible to discuss appropriate arrangements. In case of approved absence from an exam, a deferred exam will be written. For brief or temporary absences, UM students should consult the current procedures linked from [Governing Documents: Students](#). Students whose circumstances affect a substantial portion of the course should also contact their home faculty and, where appropriate, their institution's accessibility service.

Final grades

For students registered at UM, the final letter grade will be no worse than the grade determined by the following scale.

Letter grade	A+	A	B+	B	C+	C	D
Minimum percentage	95	85	78	72	66	60	50

For students registered at another university, grades will be reported in accordance with the arrangements and grading requirements of the home institution. The assessment components and weights above apply to all participants.

Tentative teaching schedule

The schedule may be adjusted to accommodate the class's background and interests, as well as any suitable new developments. Guest lectures and student presentations will be integrated into the schedule below.

Dates/block	Topics and principal sources
Jan. 7, 12, 14, 19	Covering Euclidean space. Basic notions of convex geometry; Rogers' theorem on covering space by translates of a convex body; lattice and spherical coverings; discretization; fractional and hypergraph coverings. [18, 19, 20].
Jan. 21, 26, 28; Feb. 2	The illumination conjecture. The Hadwiger–Boltyanski conjecture. New bounds for the illumination problem via discretization and additive combinatorics. [10, 4, 12, 23].
Feb. 4, 9, 11	Borsuk's question. Partitioning into sets of smaller diameter; the Frankl–Wilson theorem and the Kahn–Kalai counterexample. [11, 14, 15].
Feb. 15–19	Winter Term Break — no classes.
Feb. 23, 25; Mar. 2, 4	Bodies of constant width. Structural properties; completion of bounded sets without increasing diameter; Schramm's illumination theorem and upper bounds for Borsuk's numbers. [17, 22].
Mar. 9, 11	Volume of bodies of constant width. Volume bounds; the Blaschke–Lebesgue problem and Meissner bodies; constructions with small volume; the spherical Blaschke–Lebesgue problem. [6, 7, 13].
Mar. 16, 18	Lower bounds on illumination and covering. Exponential lower bounds for bodies of constant width; Grünbaum's problem of covering diameter-1 sets by balls of diameter 1; symmetric configurations. [1, 3].
Mar. 23, 25	Probabilistic constructions of sets that are hard to cover. A general framework in terms of measurable graphs. [5].
Mar. 30; Apr. 1, 6, 8	Universal covers and further topics. Universal covers for sets of diameter 1; known constructions; Jung's theorem; sharp asymptotic lower bounds on the volume of universal covers and applications to Borsuk's question. [5, 16]. Recent approaches to improving the exponential upper bound for Borsuk's number [24]. <i>Optional extension:</i> illumination of cap bodies via ball packing and covering estimates [2, 8, 9]. Student presentations and selected additional topics as time permits.

Important Winter 2027 dates

The dates below are from the University of Manitoba's [Important Dates and Deadlines](#) page. Students are responsible for checking the official page for changes and for dates applying to irregularly scheduled courses. Students registered at other universities must also check their home institution's administrative deadlines.

January 7	First day of UM Winter Term classes and first meeting of this course
February 15–19	Winter Term Break; no lectures
March 22	Voluntary withdrawal deadline for UM Winter Term classes
April 8	Last scheduled Tuesday/Thursday meeting of this course

Academic integrity and use of generative AI

Students are responsible for knowing and following the University of Manitoba's academic-integrity rules, including the Student Discipline Bylaw and Student Academic Misconduct Procedure. Plagiarism, unauthorized collaboration, impersonation, contract cheating, falsification, and unauthorized or undeclared technological assistance may constitute academic misconduct and may result in serious penalties. See UM's current [Academic Integrity](#) page and [Governing Documents: Students](#). Students registered at another university must also follow their home institution's academic-integrity rules and procedures. If uncertain about what is permitted, ask the instructors before submitting work.

Exams. No external aids, notes, or electronic devices are permitted during the in-person tests, except as required by an approved accommodation. This includes generative AI tools. Local supervisors will arrange proctoring for students at participating institutions.

Presentation. Generative AI tools may be used solely as a preliminary research aid. They may not directly write, derive, or generate the mathematical content or text of the presentation. Any permitted use must be disclosed with the tool, purpose, and relevant prompts or outputs. Students remain responsible for verifying and being able to explain every presented claim. AI tools may not be used in real time to assist with delivery or answers during the question-and-answer portion of the presentation.

Discussion of course ideas with classmates is encouraged, but assessed work must represent the student's own understanding. The course's discussion of research involving AI-assisted proofs does not alter these assessment rules.

Use of technology and recording of classes

Technology resources must be used responsibly, efficiently, ethically, and legally. In class, computing and communication devices should be used for course-related work or an approved accommodation, not for personal entertainment. See the applicable policies through [Governing Documents: Students](#).

The instructors and the University of Manitoba hold copyright in course materials, presentations, and lectures. The instructor-made lecture recordings described above are provided to enrolled students for private study and research. Students may not make additional audio or video recordings, or distribute recordings of lectures, office hours, or student presentations, openly or surreptitiously, in whole or in part, without prior permission from the instructors and any other affected participants. Approved accessibility accommodations apply.

Crowdmark

Your personal information is being collected under the authority of The University of Manitoba Act. It will be used for the purposes of grading papers and providing feedback to students. Personal information will not be used or disclosed for other purposes, unless permitted by The Freedom of Information and Protection of Privacy Act (FIPPA). The University of Manitoba has taken steps to ensure that its

agreement with Crowdmark, Inc. for services provided by the Crowdmark application is in compliance with FIPPA. Please be aware that information held by Crowdmark Inc. may be transmitted to and stored on servers outside of the University of Manitoba, or Canada. The University of Manitoba cannot and does not guarantee protection against the possible disclosure of your data including, without limitation, against possible secret disclosures of data to a foreign authority in accordance with the laws of another jurisdiction. If you have any questions about the collection of personal information, contact the Access and Privacy Office (tel. 204-474-9462), The University of Manitoba, 233 Elizabeth Dafoe Library, Winnipeg, Manitoba, Canada, R3T 2N2.

Accessibility and respectful learning

Student Accessibility Services (SAS) supports students with disabilities and helps remove barriers to equal access. Students who require, or think they may require, an accommodation are encouraged to contact SAS as early as possible through [Accessibility for Students](#). Approved accommodations apply to written and oral assessments; the instructors and SAS will determine an accessible format that preserves the relevant learning objectives. Students at other universities should contact their home institution's accessibility service and coordinate approved accommodations with the instructors and local course coordinator.

Every member of the class is entitled to a collegial, inclusive, and respectful learning environment. Students are expected to follow the University's [Respectful Work and Learning Environment policy](#). Support and information concerning sexual violence are available from the [Sexual Violence Resource Centre](#). Students at other universities may also use the corresponding services at their home institution.

Grades, appeals, copyright, and student supports

Students who have questions about a mark should first discuss the matter with the instructors. For UM students, information about term-work and final-grade appeals, procedures, fees, and deadlines is available from the Registrar's Office: [Appeal a Grade](#). Students registered elsewhere should consult their home institution's appeal procedures.

Copyright and intellectual-property rights must be respected. Course notes, slides, recordings, exercise and test questions, solutions, and other materials may not be posted or distributed without authorization. See the UM Copyright Office's [Copyright Support for Students](#).

Student Advocacy offers confidential help in understanding University processes, rights, responsibilities, complaints, and appeals: [Student Advocacy](#). The University's [Student Supports](#) page collects current academic, accessibility, counselling, health, wellness, safety, and emergency resources. Students at other universities should also consult their home institution's student-support resources. In an immediate emergency, call 911 or the local emergency number and follow the directions applicable to your location.

References

- [1] A. Arman, A. Bondarenko, and A. Prymak. Convex bodies of constant width with exponential illumination number. *Discrete Comput. Geom.* **74** (2025), 196–202. [arXiv:2304.10418](#).
- [2] A. Arman, A. Bondarenko, A. Prymak, and D. Radchenko. On a Gallai-type problem and illumination of spiky balls and cap bodies. *Mathematika* **71** (2025), e70017. [arXiv:2408.01341](#).
- [3] A. Arman, A. Bondarenko, A. Prymak, and D. Radchenko. On Grünbaum's problem for symmetric configurations. Preprint, 2026. [arXiv:2607.22032](#).

- [4] A. Arman, A. Bondarenko, and A. Prymak. On Hadwiger’s covering problem in small dimensions. *Canad. Math. Bull.* **68** (2025), no. 4, 1239–1250. [arXiv:2404.00547](#).
- [5] A. Arman, A. Bondarenko, A. Prymak, and D. Radchenko. On asymptotic Lebesgue’s universal covering problem. *Bull. London Math. Soc.* **58** (2026), no. 10, e70514. [doi:10.1112/blms.70514](#); [arXiv:2512.04023](#).
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- [8] A. Arman, J. S. Kaire, and A. Prymak. Illumination number of 3-dimensional cap bodies. *Discrete Math.* **349** (2026), no. 7, 115025. [arXiv:2507.08712](#).
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- [10] K. Bezdek and M. A. Khan. The geometry of homothetic covering and illumination. In *Discrete Geometry and Symmetry*, Springer Proceedings in Mathematics & Statistics, vol. 234, Springer, 2018, pp. 1–30. [arXiv:1602.06040](#).
- [11] P. Frankl and R. M. Wilson. Intersection theorems with geometric consequences. *Combinatorica* **1** (1981), 357–368.
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- [14] J. Kahn and G. Kalai. A counterexample to Borsuk’s conjecture. *Bull. Amer. Math. Soc.* **29** (1993), 60–62.
- [15] G. Kalai. Some old and new problems in combinatorial geometry I: around Borsuk’s problem. In *Surveys in Combinatorics 2015*, Cambridge University Press, 2015. [arXiv:1505.04952](#).
- [16] M. Lassak. An estimate concerning Borsuk partition problem. *Bull. Polish Acad. Sci. Math.* **30** (1982), 449–451.
- [17] H. Martini, L. Montejano, and D. Oliveros. *Bodies of Constant Width: An Introduction to Convex Geometry with Applications*. Springer, 2019.
- [18] M. Naszódi. Flavors of translative coverings. In *New Trends in Intuitive Geometry*, Bolyai Society Mathematical Studies, vol. 27, Springer, 2018, pp. 335–358. [arXiv:1603.04481](#).
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- [21] R. Schneider. *Convex Bodies: The Brunn-Minkowski Theory*. 2nd edition. Cambridge University Press, 2014.
- [22] O. Schramm. Illuminating sets of constant width. *Mathematika* **35** (1988), 280–289.

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